Fallacies and Paradoxes
Believe it or not, these guys are all the same height.
FOCUS ON THE DOT IN THE CENTRE AND MOVE YOUR HEAD BACKWARDS AND FORWARDS. WEIRD HEY...
The sun and the nearest star, Alpha Centauri, are separated by empty space.

Empty space is nothing.

Therefore nothing separates the sun from Alpha Centauri.

If nothing separates the two, then they are next to one another.

Hence, Alpha Centauri and the sun are next to one another.
If a bottle and a cork together cost $1.10, and the bottle costs $1.00 more than the cork, then what does the cork cost?
There are 365 days in a year. People work 8 hours per day. So people work the equivalent of one third of 365 days, and that is 122 days.

But people work usually only week days. So this means two days off per week, for 52 weeks this makes 104 days off per year. Subtract this from 122 days and we are left only with 18 days of work per year.

But with 10 days vacation per year, and 8 days of regular holidays, that leaves zero time for work!
We know that \(1 \text{ yard} = 36 \text{ inches}\)

Divide both sides by 4 \(1/4 \text{ yard} = 9 \text{ inches}\)

The square root of both sides is \(1/2 \text{ yard} = 3 \text{ inches}\)

Something is wrong!
\[ X = X \]
\[ X^2 = X^2 \]
\[ X^2 - X^2 = X^2 - X^2 \]
\[ X(X-X) = (X+X)(X-X) \]
\[ X = X+X \]
\[ 1 = 1 + 1 \]
\[ 1 = 2 \]
Critical Thinking: Curious Numbers

• Multiply 111,111,111 by itself
Critical Thinking: Curious Numbers

• Multiply $111,111,111$ by itself
• The answer is $12345678987654321$!
And look at this symmetry:

\[
\begin{align*}
1 \times 1 &= 1 \\
11 \times 11 &= 121 \\
111 \times 111 &= 1221 \\
1111 \times 1111 &= 123421 \\
11111 \times 11111 &= 123454321 \\
111111 \times 111111 &= 12345654321 \\
1111111 \times 1111111 &= 1234567654321 \\
11111111 \times 11111111 &= 123456787654321 \\
111111111 \times 111111111 &= 12345678987654321
\end{align*}
\]
Critical Thinking: Curious Numbers

- Select a 3 digit number (say 583) then write it again (583583). Now divide this number by 7 (you get 83369)
- Notice that you have no remainder!
- Divide the last number by 11 (you get 7579), again you have no remainder.
- Finally, divide the last number by 13 (you get 583). That is the 3 digit number you started with.
(You may need a calculator to do the divisions with no mistakes!).
Critical Thinking: Curious Numbers

- Select a 3 digit number with different digits (say 462 is fine, but 292 is not).
- Reverse this number (264), then subtract the smaller number from the larger one (462−264=198).
- Now reverse the last number you got (891).
- Finally, add the last 2 numbers (198+891=1089). YOU WILL ALWAYS GET 1089 regardless of the number you start with. (Remember that Zero is a number and cannot be ignored. Also note that it is only the first and third digits that must be different from one another, that is 229 works, but 292 does not).
Fallacies

- A fallacy is a counterfeit argument: the propositions offered as premises appear to support the conclusion, but in fact do not provide any support at all.
Subjectivism

I believe/want $p$ to be true
\[ \downarrow \]
$p$ is true

The mere fact that we have a belief or desire – is being used as evidence for the truth of a proposition.

“$I$ was just brought up to believe in $X$. ”
“That may be true for you, but it isn’t true for me.”
Appeal to Majority

The majority (of people, nations, etc.) believe $p$

$p$ is true

The fallacy of **appealing to the majority** is committed whenever someone takes a proposition to be true merely because large numbers of people believe it.
Appeal to Emotion

“In your heart you know he’s right.”
Appeal to Force

• If I “persuade” you of something by means of threats, I have not given you a reason for thinking the proposition is true; I have simply scared you into thinking, or at least into saying, it is true. In this respect, the appeal to force might be regarded as a form of the appeal to emotion.
X says $p$
X says $p$

$p$ is true!
An ad hominem argument rejects or dismisses another person’s statement by attacking the person rather than the statement itself.

\[(\text{X says } p) + (\text{X has some negative trait}) \quad \Downarrow \quad p \text{ is false}\]
In the strict and literal sense, **Begging the Question**, is the use of a proposition as a premise in an argument intended to support that same proposition.

\[
p \quad \downarrow
\]

\[
p
\]

“[1] Society has an obligation to support the needy, because [2] people who cannot provide for themselves have a right to the resources of the community”.

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**Begging the Question (Circular Argument)**

In the strict and literal sense, **Begging the Question**, is the use of a proposition as a premise in an argument intended to support that same proposition.
Post Hoc

The Latin name of this fallacy is short for *post hoc ergo propter hoc*: “after this, therefore because of this.” The fallacy has to do with causality, and it has the structure:

\[
\begin{align*}
A \text{ occurred before } B \\
\downarrow \\
A \text{ caused } B
\end{align*}
\]
False Alternative

- The fallacy of **false alternatives** occurs when we fail to consider all the relevant possibilities.
Suppose I accused you of cheating on an exam. “Prove it,” you say. “Can you prove that you didn’t?” I ask – and thereby commit the fallacy of appeal to ignorance. This fallacy consists in the argument that a proposition is true because it hasn’t been proven false.
A non sequitur argument is one in which the conclusion simply does not follow from the premises; the premises are irrelevant to the conclusion (thus another name for the fallacy is “irrelevant conclusion”).

“The pedestrian had no idea which direction to go, so I ran over him.”
Optical Illusions
Circle Limit IV, by M. C. Escher.
Duck-rabbit figure.
Indian-Eskimo figure.
Boring's young-old woman figure.
Rubin's vase-profile figure.
Mach’s reversible book. The problem for the viewer is to determine if the book is open with its spine toward us or with its pages toward us.
Schröder's famous reversible staircase is a classic ambiguous figure. At first it appears to be normal and right-side up, but upon continued viewing it will suddenly invert and appear to be upside down.
Reversible cube pattern. The reversal in this figure involves the direction of the cubes. At times the black sides appear to be the tops of cubes, and at other times the bottoms. At rare moments it is possible to see the figure as a plane crossed by black and white diamond shapes.
The Frazier twisted-cord illusion

"SPIRALS OR CIRCLES?"
WHICH ONE IS THE MIDDLE PRONG?
The interrupted horizontal line illusion

The interrupted vertical line illusion

LENGTH OF LINES AB AND CD
The top hat illusion.
The Ponzo illusion

WHICH OF THE PARALLEL LINES IS SHORTER?
A size-perception contrast illusion

WHICH CENTRAL SQUARE IS LARGER?
The Poggendorff illusion

WHICH IS THE REAL LINE?
The Muller-Lyer illusion

WHICH OF THE PARALLEL LINES IS LONGER?
The Hering illusion.

The Wundt illusion.

Are the horizontal lines parallel?
Impossible cuboid. Here the impossible connections are made by the central rib of the cuboid which appears to connect the front to the back.
Impossible quadrilateral. Notice that this illusion works by means of false connections. The corners of the “quadrilateral” connect impossibly in the same way as do the angles of the Penrose impossible triangle.
The Penrose impossible triangle
The Penrose impossible staircase
The Kanizsa triangle
The Woman with Closed Eyes: Stare at this woman and her eyes will suddenly open!
**FIGURE 5-4.** MacKay’s illusion. When we stare at this image for about ten seconds we have a sense of undulating movements that become revolving movements. Using 110 subjects, MacKay found eighty-three who saw the movement take a clockwise direction, and twenty-seven who saw it move in the reverse direction, with some subjects capable of alternating. To see the direction of rotation change, MacKay recommends fixating a point to the left or the right of the center of the design.
Logical Paradoxes
In ancient Greece, a philosopher named Protagoras was said to have taught the law to a poor student named Euathlus on the condition that Euathlus would repay Protagoras as soon as the student won its first case. However, after completing his legal studies, Euathlus decided to go into politics and did not repay his teacher. Protagoras sued Euathlus for his fee.

In the courtroom, both Protagoras and Euathlus argued their own cases with impeccable logic. Protagoras argued as follows:

“If I win this suit, Euathlus must pay.
If I lose this suit, then Euathlus has won his first case.
If Euathlus wins his first case he must pay me.

Therefore win or lose Euathlus must pay”.
On the other hand, Euathlus argued with equally strong logic as follows:

“If Protagoras loses, then I do not have to pay him.
If Protagoras wins, then I have not won my first case yet.
If I have not won my first case, then I do not have to pay.
Therefore win or lose, I do not have to pay Protagoras”.

This and other logical paradoxes have been argued for centuries, some without satisfactory conclusions, thus preserving their mystery and logical beauty.
The Liar Paradox

On one side of a card is the sentence:

“The statement on the other side of this card is true.”

And on the other side of the card is the statement:

“The statement on the other side of the card is false.”
Zeno

Consider running 100 meters. First you have to travel half that distance, then half of the remainder, and then half of the remainder, and so on for an infinite number of halves, and hence you will never finish the race!

But we know we can finish the race.

(The sum of the infinite series

\[ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots \]

converges to 1).
A professor announced to his class:

“On one day of next week I will give you an unexpected exam”.

The students had no way of knowing which day of the week they will have the exam.

But one student said:

“I am not sure if there will be an exam”.

“If we do not know about the exam until we get to class that day, then he can’t give it on Friday because by class time Thursday, if we have not had the exam then we will know he will give it the next day and that will not be unexpected”.

“Now if he gives us the exam on Thursday, then similarly we would know that by Wednesday’s class. So it can’t be Thursday either, and so on the preceding day. Therefore we cannot have an unexpected exam, and since the professor speaks the truth, then we shall have no exam”.

But on Wednesday, the professor comes to class and gives an exam!

Was this unexpected?
The Infinite Hotel

If a hotel with a finite number of rooms is completely full, then a new is customer is told:

“Sorry, we are full”.

But if you imagine a hotel with an infinite number of rooms and if the hotel is again completely full, then a new customer is told:

“Sorry, we are full, but we can give you a room”!

HOW?
The Definition of Words and the Heap Paradox

We define a collection of things put together in a place as a ‘heap’. Suppose that we have a heap of about 100 sugar cubes. Now if I take a sugar cube away, I still have a heap of sugar cubes. If I continue to take away one cube at a time then we still have a heap left. But clearly when I have a few or two or one cubes left, that is not a heap, yet we started with a heap. Since there is no definition of how many things together make a heap, and since one or two or a few cubes do not make a heap, then we do not know when a heap stops being a heap.
The Barber Paradox

In a village there is only one barber, who is always clean-shaven. He shaves all village men who do not shave themselves.

WHO SHAVES THE BARBER?

(He cannot shave himself because he will violate the statement “he shaves all the village men who do not shave themselves”. And again, if he does not shave himself he then violates the stipulation that “he shaves all village men who do not shave themselves”).

Bertrand Russell, 1918
Suppose you’re on a game show, and you’re given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what’s behind the doors, opens another door, say number 3, which has a goat. He says to you, “Do you want to pick door number 2?” Is it to your advantage to switch your choice of doors?
Yes, you should switch. The first door has a 1/3 change of winning, but the second door has a 2/3 change. Here’s a good way to visualize what happened: Suppose there are a million doors, and you pick door number 1. Then the host, who knows what’s behind the doors and will always avoid the one with the prize, opens them all except door number 777,777. You’d switch to that door pretty fast, wouldn’t you?
<table>
<thead>
<tr>
<th></th>
<th>DOOR 1</th>
<th>DOOR 2</th>
<th>DOOR 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GAME 1</strong></td>
<td>AUTO</td>
<td>GOAT</td>
<td>GOAT</td>
<td>Switch and you lose</td>
</tr>
<tr>
<td><strong>GAME 2</strong></td>
<td>GOAT</td>
<td>AUTO</td>
<td>GOAT</td>
<td>Switch and you win</td>
</tr>
<tr>
<td><strong>GAME 3</strong></td>
<td>GOAT</td>
<td>GOAT</td>
<td>AUTO</td>
<td>Switch and you win</td>
</tr>
<tr>
<td><strong>GAME 4</strong></td>
<td>AUTO</td>
<td>GOAT</td>
<td>GOAT</td>
<td>Stay and you win</td>
</tr>
<tr>
<td><strong>GAME 5</strong></td>
<td>GOAT</td>
<td>AUTO</td>
<td>GOAT</td>
<td>Stay and you lose</td>
</tr>
<tr>
<td><strong>GAME 6</strong></td>
<td>GOAT</td>
<td>GOAT</td>
<td>AUTO</td>
<td>Stay and you lose</td>
</tr>
</tbody>
</table>
Two Envelopes

A paradox:

In a game show there are two closed envelopes containing money. One contains twice as much as the other. You choose one envelope and then the host asks you if you wish to change and prefer the other envelope. Should you change? You can take a look and know what your envelope contains.

Say that your envelope contains $20, so the other should have either $10 or $40. Since each alternative is equally probable then the expected value of switching is \( \frac{1}{2} \times 10 + \frac{1}{2} \times 40 \) which equals $25. Since this is more than your envelope contains, then this suggests that you should switch. This reasoning works for whatever amount you find in your envelope. So it does not matter if you looked in your envelope or not.

But your envelope is as likely to contain twice as much as the other envelope, and if someone else was playing the game and had chosen the second envelope, then the same arguments as above would suggest that that person should switch to your envelope to have a better expected value.

See the explanation on the Friends of Astronomy web site in http://www.astro.cornell.edu
The Ship of Theseus

Over a period of years, in the course of maintenance a ship has its planks replaced one by one – call this ship A. However, the old planks are retained and themselves reconstituted into a ship – call this ship B. At the end of this process there are two ships. Which one is the original ship of Theseus?
Where is the Dollar?
Three people traveling together check into a hotel and get a single room for $30. Each person contributes $10 and they give $30 to the desk clerk. Soon the clerk notices that he made a mistake and that the room costs only $25. He calls the bellman and gives him $5 and tells him to return the money to the men that had just checked in.

The bellman thinks that it is hard to divide $5 to 3 people, and since they do not expect any refund anyway, he decides to give a dollar to each one of them and pocket the remaining $2.

But there seems to be a puzzle here. Each person originally contributed $10 for a total of $30. Each person gets a refund of $1, that is each has now paid $9, for a total of $27. The bellman pocketed $2. So $27 + $2 = $29. What happened to the other dollar?
MORAL PARADOXES

- Fortunate – Misfortune

- You break a leg and in the hospital you meet a doctor you fall in love and get happily married.
MORAL PARADOXES

• Ordinary Pure Blackmail

• He threatens you to tell your wife about your affair, unless you pay him! (Blackmail is illegal).

• But threatening to tell is not illegal, and asking for money is not illegal!
MORAL PARADOXES

• Feeling Morally Bad

• Crazy gunman shoots in your direction but suddenly two men walk in front of you and they are killed. You are saved.

• You are sorry that they died, but overall you are glad you are alive.